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TECHNICAL NOTES

Experimental investigation of heat flux uniformity at thin, electrically heated metallic foils

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INTRODUCTION

IN HEAT transfer experiments, it is common to use a very thin, electric-current-carrying metal foil as a means of obtaining a uniform surface heat flux. However, in a recent paper by Tarasuk and Castle [1], it was suggested that the longitudinal Hall effect may give rise to a nonuniform distribution of the current in the direction perpendicular to the current flow. If this were true, the resulting nonuniform Ohmic dissipation would not yield the desired uniform surface heat flux.

On the basis of experiments performed in ref. [1], it was reported that the temperature distribution along the surface of the foil (in the direction perpendicular to the current flow) was not significantly influenced by the mode of heat transfer, by the foil orientation, and by various other factors. From this finding, it was concluded that the temperature distribution was governed by a nonuniform heat flux which was intrinsic to the foil.

The issues raised in ref. [1] are significant because they tend to cast doubt on the results of a sizeable number of published experimental investigations. With this concern as a motivation, a carefully planned and executed set of foil-related heat transfer experiments was undertaken. It was thought that the experiments would be most meaningful if they were performed for a case for which there was a well-established analytical solution for uniform surface heat flux. For such a case, comparisons between the experimental and analytical results would demonstrate to what degree heat flux uniformity was achieved at the surface of the current-carrying foil.

The case chosen for study was natural convection from a vertical plate with uniform surface heat flux, an exact solution for which is presented in ref. [2]. A particular feature of this case is that the predicted vertical temperature distribution along the plate surface is not symmetric about the mid-height point, which is in contrast to the symmetric temperature distributions reported in ref. [1]. The present experiments, as well as those of ref. [1], were performed in air.

The apparatus employed here was also utilized in ref. [3] in experiments on natural convection from a heated, upward-facing horizontal surface. That case does not fit the objectives of the present investigation because there are no available analytical or numerical results for the uniform surface heat flux condition. It is relevant to note, however, that the measured surface temperature distributions [3] for the horizontal orientation of the foil were significantly different from those for the vertical orientation, which stands in contrast to the orientation independence reported in ref. [1].

It is also worth noting the outcome of voltage distribution measurements made prior to the assembly of the apparatus. Fine-tipped voltage probes were traversed across the surface of the foil in the direction perpendicular to the direction of the current flow. These measurements showed strict uniformity of the voltage, indicating that the current was also uniformly distributed.

EXPERIMENTS

The description of the experimental apparatus is facilitated by reference to Figs. 1 and 2. The first of these figures is a vertical sectional view cut midway between the horizontal extremities of the apparatus, while the second shows a horizontal section midway between the vertical extremities.

The current-carrying-foil was a 0.00254-cm-thick sheet of stainless-steel shim stock. When in place in the apparatus, the exposed surface of the foil was a vertically oriented rectangle of height $H = 7.620$ cm and horizontal length equal to 25.40 cm. The relatively large horizontal : vertical aspect ratio of 3.3 was chosen to minimize the influence of possible end effects associated with the horizontal extremities of the apparatus (temperature measurements were made midway between these extremities). Previously, the exposed surface of the foil had been painstakingly handpolished using a succession of lapping compounds, terminating with 1200 grit. The radiative emissivity of the surface was measured to be 0.10.

As seen in Fig. 1, the upper and lower edges of the foil (i.e. the horizontal edges) were free. The vertical edges (i.e. at the horizontal extremities) terminated in bus bars positioned as shown in Fig. 2. Each bus bar was a two-piece aluminum assembly whose clamplike jaw grasped the foil uniformly all along its height.

The foil was instrumented on its rear face with thermocouples and voltage taps in order to obtain local heat transfer results. Small diameter wire (0.00762 cm) was used to minimize disturbances of the temperature and voltage distributions—chromel/constantan for the thermocouples and constantan for the voltage taps. The attachment of the

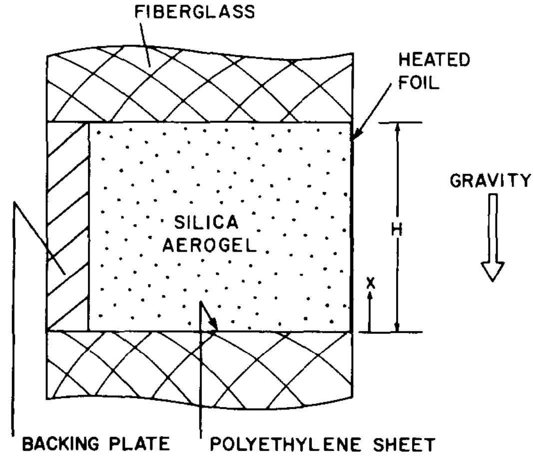

FIG. 1. Vertical sectional view cut midway between the horizontal extremities of the apparatus.

NOMENCLATURE

Gr_x^*	local modified Grashof number, equation (3)
$\overline{Gr^*}$	mean modified Grashof number, equation (8)
g	acceleration of gravity
H	height of heated metallic foil
k	thermal conductivity
q	rate of heat transfer per unit area
T_w	surface temperature
T_∞	ambient temperature
x	vertical coordinate.

Greek symbols

β	coefficient of thermal expansion
ε	emissivity
ν	kinematic viscosity
σ	Stefan–Boltzmann constant.

Subscripts

x	arbitrary vertical position
1/2	at $x/H = 1/2$.

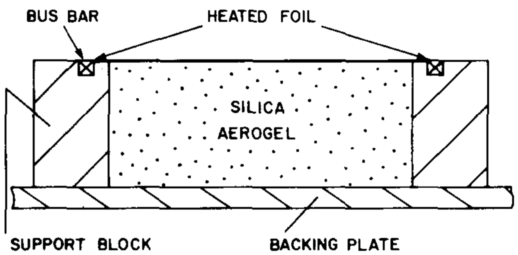

FIG. 2. Horizontal sectional view cut midway between the vertical extremities of the apparatus.

wire to the foil was accomplished with epoxy (copper-filled for the tap attachment), applied in minimal amounts. The lead wires were laid along the expected isotherms.

Thirteen of the thermocouples were deployed along a vertical line situated midway between the horizontal extremities of the foil, with additional thermocouples flanking this line to verify the absence of horizontal temperature variations. There were four voltage taps, two to either side of the aforementioned mid-line and respectively 5.08 and 10.16 cm from it.

As seen in Fig. 2, the foil was stretched between two support blocks and held fixed with epoxy. The support blocks were made of closed-pore, extruded polystyrene insulation machined with a smooth surface finish to close tolerances. A recess was milled into the front face of each block to accommodate one of the bus bars. The rear face of the block was cemented to a wooden backing plate to which an angle-iron brace (not shown) had been attached to ensure flatness.

The foil, the backing plate, and the support blocks formed four sides of a six-sided enclosure which was filled with silica aerogel powder, an insulation whose thermal conductivity is 15% less than that of air (the other sides of the enclosure were polyethylene plastic sheets). The resulting insulation bed was 9.5 cm wide in the direction normal to the foil. Thermocouples were affixed to the inboard face of the backing plate to enable evaluation of possible heat losses across the bed. Additional insulation above and below the silica aerogel bed was provided by 9-cm-thick fiberglass batts (Fig. 1).

The experiments were carried out in a laboratory which is actually a room within a room, with thermal isolation provided by 30-cm-thick cork walls. The laboratory is in a deep basement and is neither heated nor ventilated. Temperature stability was enhanced by the presence of a substantial amount of thermal mass. The power supply and other instrumentation were situated outside the laboratory, which was sealed and unlighted throughout the entire duration of a data run (about 24 h). The ambient temperature was measured by two shielded thermocouples positioned to the side of the heated foil.

Power was supplied to the foil from a DC source stable to 0.02% or better over a 24-h period. The thermocouple and current shunt voltages were read to 1 μV, while the differences between the voltage taps were read to four significant figures.

RESULTS AND DISCUSSION

The main focus of the presentation of results is a comparison of the present data with the analytical predictions of ref. [2] for the uniform surface heat flux case and with the data of ref. [1]. The temperature distributions of ref. [1] are available in terms of the ratio

$$(T_w - T_\infty)_x/(T_w - T_\infty)_{1/2} \tag{1}$$

in which T_w is the local temperature of the surface, and T_∞ is the ambient temperature. The subscript x denotes an arbitrary position along the height of the plate, while the subscript 1/2 denotes the mid-height position $x/H = 1/2$.

The present data are readily phrased in terms of equation (1), so that the main prerequisite for a graphical presentation is to cast the analytical predictions in the ratio form. From ref. [2] for $Pr = 0.7$

$$(T_w - T_\infty)_x = 2.02(qx/k)/(Gr_x^*)^{1/5} \tag{2}$$

where Gr_x^* is the so-called modified Grashof number defined as

$$Gr_x^* = g\beta qx^4/k\nu^2. \tag{3}$$

If the surface heat flux q is strictly uniform and variations of the thermophysical properties are neglected, as in the analysis of ref. [2], it follows that

$$(T_w - T_\infty)_x/(T_w - T_\infty)_{1/2} = [x/(H/2)]^{1/5}. \tag{4}$$

In any real experiment, however, q will not be strictly uniform because of radiation and conduction losses, even if there is uniform Ohmic dissipation in the foil. In addition, owing to the x-dependence of T_w, the reference temperature for the evaluation of the thermophysical properties will vary with x. These realities suggest a local interpretation of equation (2) as follows

$$(T_w - T_\infty)_x/(T_w - T_\infty)_{1/2} = (q_x/q_{1/2})[x/(H/2)] \times (k_{1/2}/k_x)(Gr_{1/2}^*/Gr_x^*)^{1/5} \tag{5}$$

which will be employed for comparison purposes in addition to equation (4).

Numerical values of the local heat flux q_x used as input to equation (5) were obtained from

$$q_x = q_{\mathrm{Ohm}} - q_{\mathrm{rad}} - q_{\mathrm{cond}}. \tag{6}$$

In this equation, q_{Ohm} is the local heat flux due to Ohmic dissipation (assumed uniform), while q_{rad} and q_{cond} are the local radiation and conduction losses given by

$$q_{\mathrm{rad}} = \varepsilon\sigma(T_w^4 - T_\infty^4)_x, \quad q_{\mathrm{cond}} = k_{\mathrm{ins}}(T_w - T_{\mathrm{back}})_x/L_{\mathrm{ins}} \tag{7}$$

where k_{ins} and L_{ins} are the thermal conductivity and bed thickness of the silica aerogel, T_{back} is the temperature of the inboard face of the backing plate, and the other symbols are standard. The thermophysical properties of air appearing in equation (5) were evaluated at a reference temperature $1/2(T_w + T_\infty)_x$.

With the foregoing as background, attention will now be turned to the results. In ref. [1], the reported temperature

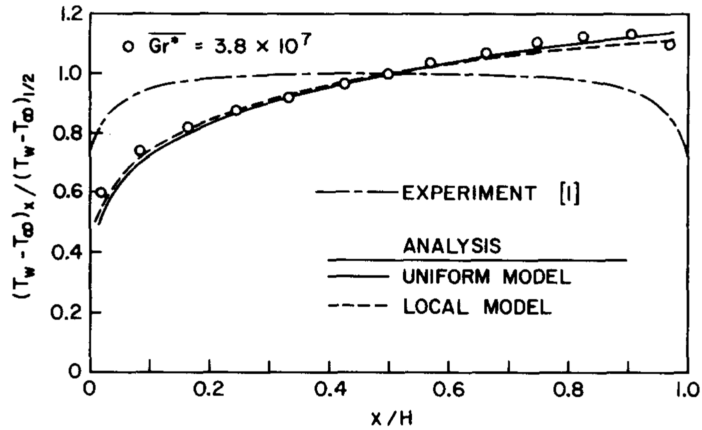

FIG. 3. Measured surface temperature distribution (heating rate $\overline{Gr^*} = 3.8 \times 10^7$) compared with analytical predictions for uniform heat flux and with experimental results from ref. [1].

distributions are parameterized by the electric current density in the foil. The magnitude of the smallest current density reported in ref. [1] is nearly equal to the largest current density of the present experiments (388 vs 418 A cm^{-2}). The results of these complementary experiments are brought together in Fig. 3 along with the analytical predictions expressed by equation (4) (uniform model) and by equation (5) (local model). In the figure, the data symbols correspond to the present results, while the results of ref. [1] are represented by the long/short dashed curve.

From the figure, it is seen that the two versions of the analytical predictions are nearly coincident, so that for comparisons with the experimental data, they may be regarded as the same. Further inspection of the figure shows that the present data are in excellent agreement with the analytical predictions (aside from localized deviations due to edge effects). This finding lends strong support to the concept that a current-carrying foil can, indeed, provide uniform Ohmic dissipation.

In contrast, the comparison of the experimentally determined temperature distribution of ref. [1] with the analytical predictions is not very satisfactory. Not only are there significant deviations in magnitude, but the shape of the experimental temperature distribution is at variance with that of the analysis. On this basis, it may be concluded that the heat flux was not uniformly distributed in the experiments of ref. [1], but there is no way of knowing what features of the apparatus caused the nonuniformity. Clearly, no such nonuniformities occurred in the present experiments.

Further comparisons between the present experiments and the analytical predictions are conveyed in Fig. 4. The feature which distinguishes the various sets of data is the heating rate, which may be represented in dimensionless terms by the mean modified Grashof number $\overline{Gr^*}$ defined as

$$\overline{Gr^*} = g\bar{q}H^4(\beta/k\nu^2). \qquad (8)$$

In this equation, $\bar{q}$ is the average value of q_x, and the thermophysical properties are evaluated at the reference temperature $1/2(\bar{T}_w + T_\infty)$.

The data displayed in Fig. 3 are characterized by $\overline{Gr^*} = 3.8 \times 10^7$, while those in Fig. 4 correspond to $\overline{Gr^*}$ values of 1.6×10^7 and 8.5×10^6.

Turning now to Fig. 4, it is seen that the results presented there fully reinforce the conclusions that were drawn from Fig. 3. In particular, for both of the heating rates considered in Fig. 4, excellent agreement prevails between the experimental data and analytical predictions which are based on uniform surface heat flux.

CONCLUDING REMARKS

Strong evidence has been presented here to support the concept that a very thin, electric-current-carrying metal foil

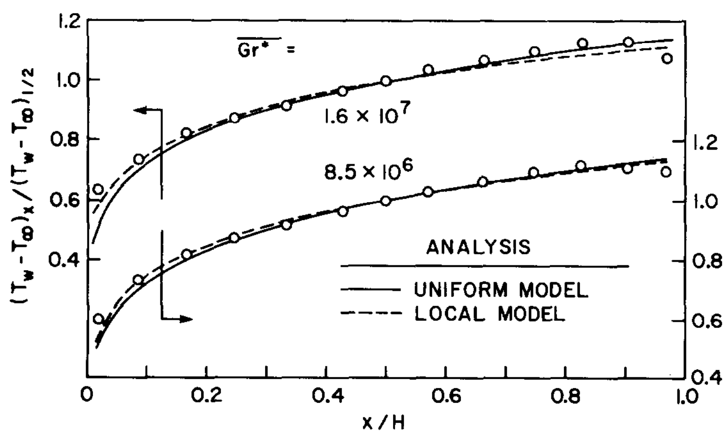

FIG. 4. Measured surface temperature distributions (heating rates $\overline{Gr^*} = 8.5 \times 10^6$ and 1.6×10^7) compared with analytical predictions for uniform heat flux.

produces uniform Ohmic dissipation and uniform surface heat flux. The evidence is embodied in the excellent agreement between the measured temperature distributions along a vertically oriented, electrically heated foil and the analytical predictions for natural convection at a vertical plate with uniform surface heat flux.

This finding stands in contrast to ref. [1], where an intrinsically nonuniform heat flux was postulated to explain measured temperature distributions that were independent of the orientation of the heated foil, the mode of heat transfer, and various other factors. In further contrast, supplementary experiments performed here [3] indicated that the surface temperature distributions were highly sensitive to whether the foil was oriented vertically or horizontally. Furthermore, local voltage measurements did not reveal nonuniformities in the distribution of the electric current that appear to have existed in ref. [1].

REFERENCES

1. J. D. Tarasuk and G. S. P. Castle, Temperature distribution in an electrically heated wide metallic foil, *J. Heat Transfer* **105**, 210–212 (1983).
2. E. M. Sparrow and J. L. Gregg, Laminar free convection from a vertical plate with uniform surface heat flux, *Trans. Am. Soc. Mech. Engrs* **78**, 435–440 (1956).
3. C. K. Carlson, Natural convection from a uniformly heated horizontal surface shrouded above by a parallel, equi-dimension plate. Thesis, Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN (1984).

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A heat balance integral method based on an enthalpy formulation

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1. INTRODUCTION

THE HEAT balance integral (HBI) method developed by Goodman [1] is a useful engineering tool which can give a quick estimate of parameters associated with a diffusion-driven phase change. Many extensions and refinements to Goodman's method have been presented. Two important examples are: Hills [2], who developed the method for metallurgical applications; and Bell [3], who coupled finite-element concepts with the Goodman technique to develop a method which can give very high accuracy.

Goodman's HBI method is based on the Stefan formulation of a one-dimensional phase change. This involves satisfying a heat balance condition at an isothermal phase-change boundary. In some ways this represents a drawback in that many practical problems have the phase change taking place over a temperature range, e.g. the solidification of a binary alloy [4]. If this so-called 'mushy' region is significant then the Goodman HBI method may not be suitable. Recently Voller [4] has developed a heat balance integral method based on an enthalpy formulation for the analysis of a binary alloy. The aim of this paper is to generalise and investigate some aspects of this technique. In particular the performance of the enthalpy heat balance integral (EHBI) will be compared with previous HBI methods in the solution of: (i) a limiting case of a mushy region solidification; and (ii) a one-dimensional, isothermal one-phase Stefan problem.

2. BASIC PRINCIPLES—A MUSHY SOLUTION

The basic principles of the EHBI can be outlined on considering the following phase-change problem. Liquid initially at temperature $T = \varepsilon$ fills the positive half space $x > 0$, the liquid is such that it undergoes a liquid/solid phase change between temperatures $T = \varepsilon$ and $T = -\varepsilon$ with a loss of latent heat L. At time $t = 0$ the surface temperature at $x = 0$ is lowered to a temperature $T = T_0 < -\varepsilon$ in order that the phase change commences. The state of the system at time $t > 0$ is shown in Fig. 1. If the thermal properties are constant and heat conduction is taken as the only mechanism of heat transfer the following governing equations may be derived,

$$\frac{\partial}{\partial t}[T+H/C] = \kappa\frac{\partial^2 T}{\partial x^2} \tag{1}$$

the enthalpy formulation, where

$$H/C = \begin{cases} 0 & -\varepsilon > T \\ F(T)\alpha & -\varepsilon \leqslant T \leqslant \varepsilon \\ \alpha & T > \varepsilon \end{cases} \tag{2}$$

$\alpha = L/C$ and $F(T)$ is some function of temperature T which determines the nature of the phase change in the mushy region. Note that $F(\varepsilon) = 1$ and $F(-\varepsilon) = 0$.

The basic HBI approach is to approximate the temperature profile in the intervals $[0, X_0]$ and $[X_0, X_1]$ (see Fig. 1) by the quadratic profiles

$$\begin{aligned} U_0 &= T_0 - \frac{x}{X_0}(T_0+\varepsilon) + a_0 x\left[1-\frac{x}{X_0}\right] \\ U_1 &= -\varepsilon + 2\varepsilon\frac{(x-X_0)}{X_1-X_0} + a_1(x-X_0)\left[1-\frac{(x-X_0)}{(X_1-X_0)}\right]. \end{aligned} \tag{3}$$

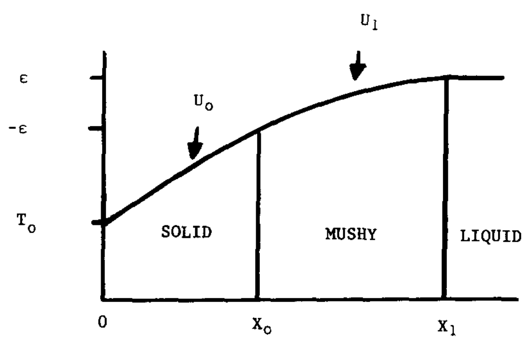

FIG. 1. State of freezing system at time t and showing approximating profiles.